Lecture 9

Friday, September 23, 2016 9:26 AM

OFFICE HOURS CANCELLED TODAY.

Higher Order Derivatives

f is a diff func, f' is also a func, so f' may or may not have a derivative, but if it does, then it's derivative $(f')' = f''$ is called the second derivative of f.

If $y = f(x)$, $f''(x) = \frac{d^2y}{dx^2}$

Similarly the 3rd derivative of f is the derivative of the 2nd derivative, $f''' = (f'')'$

$y = f(x)$, $f''' = \frac{d^3y}{dx^3}$

In general, the n^{th} derivative of $y = f(x)$ is denoted by

$$f^{(n)}(x) = \frac{d^ny}{dx^n}$$

- $f^{(9)}(x) \equiv 9^{th}$ derivative of f

Ex Find the first, second & third

derivative of $f(\theta) = e^{\theta}\cos\theta$.

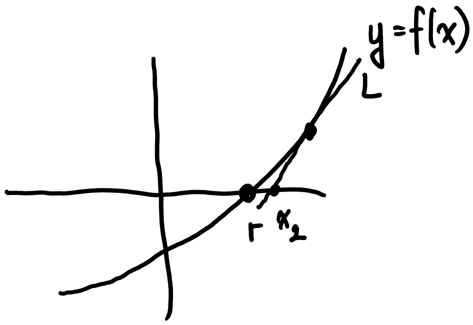
$$f'(\theta) = e^{\theta}\cdot\cos\theta + e^{\theta}\cdot(-\sin\theta)$$
$$= e^{\theta}\cdot\cos\theta - e^{\theta}\sin\theta$$

$$f''(\theta) = e^{\theta}\cdot\cos\theta - e^{\theta}\sin\theta$$
$$- \left(e^{\theta}\sin\theta + e^{\theta}\cos\theta\right)$$
$$= -2e^{\theta}\sin\theta$$

$$f'''(\theta) = -2\left[e^{\theta}\sin\theta + e^{\theta}\cos\theta\right]$$
$$= -2e^{\theta}\left[\sin\theta + \cos\theta\right]$$

4.8 Newton's Method

GOAL Solve an equation of the form $f(x) = 0$.



1) Pick a first approximation x_1.
(May be guess, or rough sketch, or table)

2) Consider the tangent Line L to the curve $y = f(x)$ at pt $(x_1, f(x_1))$ and Look at it's x-intercept, call it x_2.

3) Find x_2.

$\boxed{y - f(x_1) = f'(x_1)(x - x_1)}$: Eq^n of L.

$(x_1, f(x_1))$ w/ slope $f'(x_1)$

Since x intercept of L is x_2 means L passes through $(x_2, 0)$.

Then, $0 - f(x_1) = f'(x_1)(x_2 - x_1)$

$$\Rightarrow \frac{-f(x_1)}{+f'(x_1)} = x_2 - x_1$$

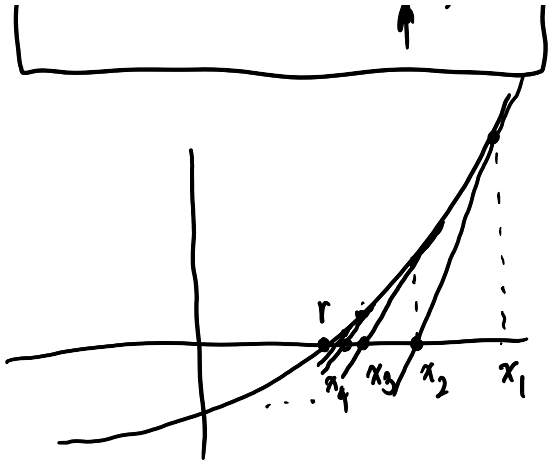
$$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

4) Repeat process for $\underline{(x_2, f(x_2))}$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

Keep going $x_1, x_2, x_3, x_4, \ldots$

$$\boxed{\underline{\underline{x_{n+1}}} = x_n - \frac{f(x_n)}{f'(x_n)}} \rightarrow$$



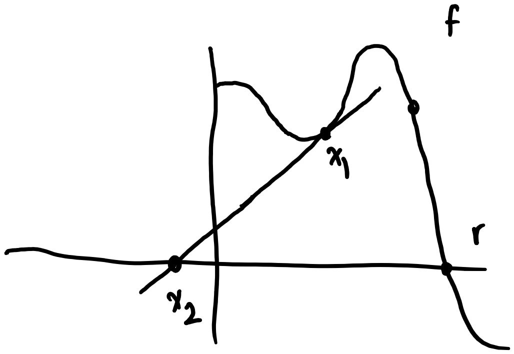
If the numbers x_n become closer and closer to r as n becomes larger & larger we say $\lim_{n\to\infty} x_n = r$.

(Sequence converges)

<u>Newton's Method</u> The sequence of approximation converges to r.

<u>RmK</u> Certain cases sequence of approximation doesn't exist or doesn't converge.

In such cases, A better x_1 should be chosen.



Ex i) Show that $\cos x = x$ has a solution between 0 & 1.

ii) Then find the root of the equation, correct to 6 decimal places.

i) $f(x) = \cos x - x$ is continuous on $[0,1]$ and use I.V.T. (DIY)

x	$\cos x - x$
0.2	
0.4	
0.6	+
0.8	−
1.0	

ii) Want to use Newton's method:

x_1

$f(x) = \cos x - x$

$f'(x) = -\sin x - 1$

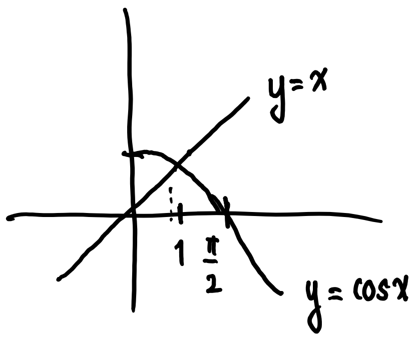
0.7

From above, $\boxed{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}}$

$$x_{n+1} = x_n - \frac{\cos x_n - x_n}{-\sin x_n - 1}$$

$$= x_n + \frac{\cos x_n - x_n}{\sin x_n + 1} \checkmark$$

GUESS A SUITABLE VALUE FOR x_1



Graph them in Maple.

Use $x_1 = 1$ as our first approximation.

$x_1 = 1$

$$x_2 = x_1 + \frac{\cos x_1 - x_1}{\sin x_1 + 1} = 1 + \frac{\cos 1 - 1}{\sin 1 + 1} \approx 0.75036837$$

$$x_3 = x_2 + \frac{\cos x_2 - x_2}{\sin x_2 + 1} \approx 0.\underline{73911}289$$

$x_4 \approx 0.\underline{739085}13$

$x_5 \approx \underline{0.739085}13$

The root is 0.739085 (up to 6 decimal places)